Robust load-frequency control for uncertain nonlinear power systems: A fuzzy logic approach

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Abstract

A new robust load-frequency control (LFC) methodology is proposed for controlling uncertain nonlinear power systems. Critical nonlinearity in the power system—the valve position limit on the governor, and the parametric uncertainty are concerned. The Takagi–Sugeno fuzzy model of the power system under consideration is first constructed to design the robust fuzzy-model-based LFC. Sufficient conditions for the robust asymptotic convergence of the frequency deviation are then provided in terms of linear matrix inequalities. Boundedness of the other system variables is also studied to ensure justifiable grounds for use of the proposed LFC method. Simulation results convincingly validate the effectiveness of the novel LFC design scheme and the theoretical discussions, which give a positive answer to the quality control of the electric energy. © 2006 Elsevier Inc. All rights reserved.
Keywords: Load-frequency control; Fuzzy-model-based control; Valve position limits; Parametric uncertainties; Uniformly ultimately boundedness

1. Introduction

The more industry and commerce grow, the more reliable electric energy with good quality, which is evaluated in terms of a frequency and a rated power, etc., is required. The introduction of the deregulation policy to power system operation makes the load-frequency control (LFC) be refocused because the repeatedly deregulated power generation under the inevitable electricity market game provokes large deviation from the standard frequency of power systems. During the last three decades, various control strategies for LFC have been proposed [1–6]. In the literature, it is commonly conjectured that, since a normally operated power system is only exposed to small change in the vicinity of the load demand, a linearized model is enough to express the dynamic behavior of the system around the operating point.

However, when a sudden large change in the load demand occurs by deregulated operations, frequent on–off controls of large capacity load units may cause large amount of overshoot or long-lasting oscillation on the valve position of the governor [4]. Therefore, it is necessary to consider the limits on the valve position for avoiding large overshoot and oscillation, or due to the mechanical characteristics, which means that the system is nonlinear. Hence, contrary to the previous theoretical point of view, the linearized model is not a good representation of the power system. Furthermore, existing LFC schemes based on the linearized model may not effectively achieve the LFC objective. So far, there have been few research works tackling on the LFC issue considering the valve position limits. Recently, Moon et al. [4,5] studied an LFC scheme considering the valve position limits. However, they used it in simulation only. This nonlinearity must be taken into theoretical consideration in the LFC design procedure to promise high power quality.

On the other hand, it is occasionally very difficult to obtain the accurate values of some parameters of the power systems. This is due to the inaccurate measurement or on-line variation of parameters, and definitely influences on the stability of the power system. Therefore, robustness against the parametric uncertainties should also be secured in the LFC problem.

Nonlinearities from the valve position limits and the parametric uncertainties weigh the stability analysis and the LFC design down with additional difficulties. Until now, various control techniques have been developed for uncertain nonlinear systems. Among them, the Takagi–Sugeno (T–S) fuzzy-model-based control technique is very popular today because it is highly regarded as a powerful resolution to bridge the gap between the fuzzy logic
and the fruitful linear control theories. The fuzzy-model-based control technique has shown many promising results that may lay a solid foundation for the control of complex nonlinear systems [7–9]. We anticipate that this can be successfully applied to the LFC problem considered in this paper as well.

Motivated by the above observations, this paper aims at resolving the LFC problem for power systems subject to the nonlinearity—the valve position limits, and the parametric uncertainties, using fuzzy logic. The presented results should be expressly discriminated from the previous works in that the concept of the valve position limits is theoretically incorporated into the design procedure. The main contributions of this paper are twofold: (1) fuzzy modeling of the nonlinear power system with the valve position limits; (2) derivation of some sufficient conditions for accomplishing the objective of the LFC maintaining the robustness against the nonlinearities and the uncertainties. Specifically, the Lyapunov stability criterion is employed to design a suitable fuzzy-model-based control supposing that the external disturbance is zero. In fact, the incremental change of the load demand, which is usually assumed as a step function, can be effectively canceled out by the integral control technique. However, the only expectable thing is the asymptotic convergence of the frequency deviation. Hence, the rigorous examination of dynamic behaviors of the other system variables is studied, which is another minor contribution of this paper.

This paper is organized as follows: the T–S fuzzy system is briefly reviewed in Section 2. In Section 3, the T–S fuzzy model for the nonlinear power system considering the valve position limits in the governor is developed. The robust fuzzy-model-based LFC synthesis conditions are proposed in Section 4. Section 5 shows simulation results. Finally, concluding remarks are drawn in Section 6.

2. Preliminaries

Many physical systems are very complex in practice so that rigorous mathematical models can be difficult to obtain. Fortunately, a certain class of nonlinear systems can be expressed as an aggregation of linear mathematical models. The T–S fuzzy system is a convenient tool in designing a control for such a nonlinear system.

Consider a nonlinear system of the following form:

\[ \dot{x}(t) = f(x(t), u(t), w(t)) \]  

(1)

where \( x(t) \in \mathbb{R}^n \) the state; \( u(t) \in \mathbb{R}^m \) the control input; and \( w(t) \in \mathbb{R}^p \) the external disturbance. The vector field \( f : U \subset \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p \rightarrow V \subset \mathbb{R}^n \) on a compact set \( U \) is assumed to be affine in \( u(t) \) and \( w(t) \), respectively, and \( C^r \)
(r ≥ 1). One way to view the T–S fuzzy system is that it performs a nonlinearly interpolated linear mapping \( \phi: U \rightarrow V \) so as to satisfy

\[
\sup_{x(t), u(t), w(t) \in U} \| f(x(t), u(t), w(t)) - \phi(x(t), u(t), w(t)) \| \leq \varepsilon
\]

where \( \varepsilon \in \mathbb{R}_{>0} \) is an arbitrary small scalar.

Assume there exist \( q \) triplets \( v_i = (A_i, B_i, D_i) \) which represent the local dynamic behavior of (1), such that the matrix polytope \( \mathcal{F} = \text{Co}\{[A_1, B_1, D_1], \ldots, [A_q, B_q, D_q]\} \) contains the domain \( U \), where \( \text{Co} \) denotes a convex hull of the set \( V = \{v_1, \ldots, v_q\} \), and \( A_i \in \mathbb{R}^{n \times m}, B_i \in \mathbb{R}^{n \times m} \), and \( D_i \in \mathbb{R}^{n \times p} \). Thus, one can find an adequate mapping at time instant \( t \) with \( \varepsilon \) of the form:

\[
\phi(x(t), u(t), w(t)) = A(\theta)x(t) + B(\theta)u(t) + D(\theta)w(t)
\]

where \( A(\theta) \) ranges over a matrix polytope \( A(\theta) \in \text{Co}\{A_1, \ldots, A_q\} \), and \( B(\theta) \in \text{Co}\{B_1, \ldots, B_q\}, \ D(\theta) \in \text{Co}\{D_1, \ldots, D_q\} \) with \( \sum_{i=1}^{q} \theta_i = 1, \quad \theta_i \geq 1, \quad i \in \mathcal{I}_q = \{1, 2, \ldots, q\} \). The key idea of the T–S fuzzy system is to determine the coefficients \( \theta_i \) in the convex combination of the given vertices \( V \) by virtue of the qualitative knowledge available from domain experts, which is quantified by ‘IF–THEN’ rule base. More precisely, the \( i \)th rule of the T–S fuzzy system is formulated in the following form:

\[
R^i: \text{IF } z_1(t) \text{ is } \Gamma^i_1 \text{ and } \cdots \text{ and } z_p(t) \text{ is } \Gamma^i_p \quad \Rightarrow \quad \dot{x}(t) = A_ix(t) + B_iu(t) + D_iw(t)
\]

where \( R^i \) denotes the \( i \)th fuzzy inference rule, \( z_k(t), \ h \in \mathcal{I}_p = \{1, 2, \ldots, p\} \), is the premise variable, and \( \Gamma^i_h \) is the fuzzy set of the \( h \)th premise variable in the \( i \)th fuzzy inference rule.

Generally speaking, in many cases it is very difficult, if not impossible, to obtain the accurate value of some system parameters. Hence, setting the consequent part of the fuzzy inference rules as an uncertain linear system allows us to deal with a wider class of nonlinear systems.

\[
R^i: \text{IF } z_1(t) \text{ is } \Gamma^i_1 \text{ and } \cdots \text{ and } z_p(t) \text{ is } \Gamma^i_p \quad \Rightarrow \quad \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + (D_i + \Delta D_i)w(t)
\]

where \( \Delta A_i, \Delta B_i \) and \( \Delta D_i \) are uncertain matrices.

By using a fuzzy inference with a singleton fuzzifier, product inference, and center average defuzzifier, the global output of (3) is represented as follows:

\[
\dot{x}(t) = \sum_{i=1}^{q} \theta_i(z(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + (D_i + \Delta D_i)w(t))
\]

where \( \theta_i(z(t)) = \frac{\xi_i(z(t))}{\sum_{i=1}^{q} \xi_i(z(t))} \), \( \xi_i(z(t)) = \prod_{h=1}^{n} \Gamma^i_h(z_h(t)) \), and \( \Gamma^i_h(z_h(t)) \) is the grade of membership of \( z_h(t) \) in the fuzzy set \( \Gamma^i_h \). Since the uncertainties in (4) may
be time-varying, it is not easy to manage. Hence we introduce the following assumption.

**Assumption 1.** The uncertainties are norm-bounded of the form:

\[
\begin{bmatrix}
\Delta A_i \\
\Delta B_i \\
\Delta D_i
\end{bmatrix} = H_i F_i(t) [E_{a_i} E_{b_i} E_{d_i}]
\]

where \(F_i(t)\) is an unknown matrix function with Lebesgue-measurable elements and satisfies \(F_i(t)^T F_i(t) \leq I\), in which \(H_i, E_{a_i}, E_{b_i}, \text{ and } E_{d_i}\) are known real constant matrices of appropriate dimensions.

3. Nonlinear power system and its fuzzy modeling

In this section, two-area nonlinear power system with the valve position lim-its and its T–S fuzzy model are discussed for designing the fuzzy-model-based LFC.

Taking the equations for the power equilibrium, the change in generation with the non-reheat type turbine, position of the governor, in the \(k\)th area system and the incremental tie-line flow, we have

\[
\begin{align*}
\Delta f_k(t) &= -\frac{1}{T_{P_k}} \Delta f_k(t) + \frac{K_{P_k}}{T_{P_k}} \Delta P_{G_k}(t) - \frac{K_{P_k}}{T_{P_k}} \Delta P_{T_k}(t) \\
\Delta P_{G_k}(t) &= -\frac{1}{T_{G_k}} \Delta P_{G_k}(t) + \frac{1}{T_{G_k}} \Delta X_{G_k}(t) \\
\Delta X_{G_k}(t) &= -\frac{1}{R_{G_k} T_{G_k}} \Delta f_k(t) - \frac{1}{T_{G_k}} \Delta X_{G_k}(t) + \frac{1}{T_{G_k}} \Delta P_{C_k}(t) \\
\Delta P_{T_{kl}}(t) &= T_{kl} \Delta f_k(t) - T_{kl} \Delta f_i(t)
\end{align*}
\]

where \(k \in \mathcal{J}_2 = \{1, 2\} \text{ and } l \in \mathcal{J}_2 \setminus \{k\} \). The description of the system variables and parameters is given in Table 1. The assumption on \(\Delta P_{D_k}(t)\) widely used in the literature [1–6] is adopted as follows:

**Assumption 2.** Throughout this paper, \(\Delta P_{D_k}(t)\) is assumed to be a step function and to satisfy

\[
\|\Delta P_{D_k}(t), \Delta P_{D_k}(t)\| \leq \varsigma
\]

where \(\varsigma \in \mathbb{R}_{\geq 0}\) is a constant.

To mitigate the effect of \(\Delta P_{D_k}(t)\) on the asymptotic convergence of \(\Delta f_k(t)\), an integral control is conventionally applied, i.e.,

\[
\Delta \dot{E}_k(t) = K_{E_k} (\Delta f_k(t) + \Delta P_{T_k}(t))
\]
Eqs. (5) and (6) are widely used in the LFC problem since it has been commonly conjectured that a power system is only exposed to small changes in the load demand during its normal operation.

**Remark 1.** In case of large change in $\Delta P_{D_k}(t)$, (5) is no longer a good approximation because the valve position on the governor is limited for the practical reasons such as the avoidance of the large overshoot or the mechanical limits of the valve position in the governor [4,5]. When a large $\Delta P_{D_k}(t)$ occurs, no matter how one increases $\Delta P_{C_k}(t)$ to regulate $\Delta f_k(t)$, $\Delta X_{G_k}(t)$ will not increase beyond a given level. In order to develop the LFC scheme that ensures the stability of the power system with the valve position limits, this nonlinearity should be theoretically handled with care.

The valve position limits can be modeled in several ways [4]. Among them, the practical piston-like steam valve structure shown in Fig. 1 is adopted in this study. The outcome limiter representing its cut-off action is shown in Fig. 2, where $\Delta X_{GM_k}$ and $\Delta X_{Gm_k}$ denote the open and the close limits of $\Delta X_{G_k}$, respectively.

Choosing

$$x(t) = [\Delta f_1(t), \Delta P_{G_1}(t), \Delta X_{G_1}(t), \Delta E_1(t), \Delta P_T(t), \Delta f_2(t), \Delta P_{G_2}(t), \Delta X_{G_2}(t), \Delta E_2(t)]^T$$

and $u(t)$ as $[\Delta P_{C_1}(t), \Delta P_{C_2}(t)]^T$, $w(t)$ as $[\Delta P_{D_1}(t), P_{D_2}(t)]^T$, and including the valve position limits, the vector field of the considered system is represented as follows:

---

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta f_k(t)$</td>
<td>Incremental frequency deviation</td>
</tr>
<tr>
<td>$\Delta P_{G_k}(t)$</td>
<td>Incremental change in the generator output</td>
</tr>
<tr>
<td>$\Delta X_{G_k}(t)$</td>
<td>Incremental change in the governor valve position</td>
</tr>
<tr>
<td>$\Delta P_{D_k}(t)$</td>
<td>Incremental change in the load demand</td>
</tr>
<tr>
<td>$\Delta P_{C_k}(t)$</td>
<td>Incremental change in the speed changer position</td>
</tr>
<tr>
<td>$\Delta P_{T_k}(t)$</td>
<td>Incremental tie-line flow</td>
</tr>
<tr>
<td>$R_k$</td>
<td>Speed regulation parameter</td>
</tr>
<tr>
<td>$T_{G_k}$</td>
<td>Time constant of the governor</td>
</tr>
<tr>
<td>$T_{T_k}$</td>
<td>Time constant of the turbine</td>
</tr>
<tr>
<td>$T_{P_k}$</td>
<td>Time constant of the plant</td>
</tr>
<tr>
<td>$K_{P_k}$</td>
<td>Plant gain</td>
</tr>
<tr>
<td>$K_{E_k}$</td>
<td>Integral control gain</td>
</tr>
<tr>
<td>$T_{kl}$</td>
<td>Maximum tie-line flow between area $k$ and $l$</td>
</tr>
</tbody>
</table>
The block diagram of (7) is sketched in Fig. 3. The system (7) has two nonlinear terms, $\eta_1(x_3(t))$ and $\eta_2(x_8(t))$. If these nonlinear terms can be represented as convex combinations of appropriate vertices, the analytic T–S fuzzy system is constructed. Assume $x_3(t) \in [\Omega_1^L, \Omega_1^U]$ and consider the following equations:

$$
\dot{x}(t) = \begin{bmatrix}
-\frac{1}{T_{p_1}} x_1(t) + \frac{K_{p_1}}{T_{p_1}} x_2(t) - \frac{K_{p_1}}{T_{p_1}} x_5(t) - \frac{K_{p_1}}{T_{p_1}} w_1(t) \\
-\frac{1}{T_{c_1}} x_2(t) + \frac{1}{T_{c_1}} \eta_1(x_3(t)) \\
-\frac{1}{R_{i_1}} x_1(t) - \frac{1}{R_{i_1}} x_3(t) - \frac{1}{R_{i_1}} u_4(t) + \frac{1}{R_{i_1}} u_1(t) \\
K_{E_1} x_1(t) + K_{E_1} x_5(t) \\
T_{12} x_1(t) - T_{12} x_6(t) \\
\frac{K_{p_2}}{T_{p_2}} x_5(t) - \frac{1}{T_{p_2}} x_6(t) + \frac{K_{p_2}}{T_{p_2}} x_7(t) - \frac{K_{p_2}}{T_{p_2}} w_2(t) \\
-\frac{1}{T_{c_2}} x_7(t) + \frac{1}{T_{c_2}} \eta_2(x_8(t)) \\
-\frac{1}{R_{i_2}} x_6(t) - \frac{1}{R_{i_2}} x_8(t) - \frac{1}{R_{i_2}} x_9(t) + \frac{1}{R_{i_2}} u_2(t) \\
-K_{E_2} x_5(t) + K_{E_2} x_6(t)
\end{bmatrix}
$$

(7)

The block diagram of (7) is sketched in Fig. 3. The system (7) has two nonlinear terms, $\eta_1(x_3(t))$ and $\eta_2(x_8(t))$. If these nonlinear terms can be represented as convex combinations of appropriate vertices, the analytic T–S fuzzy system is constructed. Assume $x_3(t) \in [\Omega_1^L, \Omega_1^U]$ and consider the following equations:

$$
\eta_1(x_3(t)) = \Gamma_1^1(x_3(t)) \cdot x_3(t) + \Gamma_1^2(x_3(t)) \cdot x_1 x_3(t)
$$

(8)

$$
1 = \Gamma_1^1(x_3(t)) + \Gamma_1^2(x_3(t))
$$

(9)
Plant rules

\[ R^1: \text{IF} \ x_3(t) \text{ is } I_1^1 \text{ and } x_8(t) \text{ is } I_1^2 \text{ THEN } \dot{x}(t) = A_1x(t) + B_1u(t) + D_1w(t) \]

\[ R^2: \text{IF} \ x_3(t) \text{ is } I_2^1 \text{ and } x_8(t) \text{ is } I_1^2 \text{ THEN } \dot{x}(t) = A_2x(t) + B_2u(t) + D_2w(t) \]

\[ R^3: \text{IF} \ x_3(t) \text{ is } I_1^1 \text{ and } x_8(t) \text{ is } I_2^2 \text{ THEN } \dot{x}(t) = A_3x(t) + B_3u(t) + D_3w(t) \]

\[ R^4: \text{IF} \ x_3(t) \text{ is } I_2^1 \text{ and } x_8(t) \text{ is } I_2^2 \text{ THEN } \dot{x}(t) = A_4x(t) + B_4u(t) + D_4w(t) \]

where

\[ \alpha_1 = \min \left( \frac{\Delta X_{GM1}}{\Phi_1}, \frac{\Delta X_{GM2}}{\Phi_2} \right) \]

Solving (8) and (9) gives

\[
\begin{align*}
\Gamma_1^1(x_3(t)) &= \eta_1(x_3(t)) - \alpha_1 x_3(t) \\
\Gamma_1^2(x_3(t)) &= \frac{x_3(t) - \eta_1(x_3(t))}{1 - \alpha_1 x_3(t)}
\end{align*}
\]

Applying the same procedure to \( \eta_2(x_8(t)) \), we have \( \Gamma_1^2(x_8(t)) = \frac{\eta_2(x_8(t)) - \alpha_2 x_8(t)}{1 - \alpha_2 x_8(t)} \) and \( \Gamma_2^2(x_8(t)) = \frac{x_8(t) - \eta_2(x_8(t))}{(1 - \alpha_2 x_8(t))} \) where \( \alpha_2 = \min \left( \frac{\Delta X_{GM3}}{\Phi_3}, \frac{\Delta X_{GM4}}{\Phi_4} \right) \). A nominal T–S fuzzy system is then modeled as follows:

Fig. 3. Block diagram of the power system with the valve position limits, \( k \in \mathcal{F}_2, l \in \mathcal{F}_2 \setminus \{k\} \).
and \( D_i = \left[ -\frac{K_{P_{1i}}}{T_{P_{1i}}} 0 0 0 0 -\frac{K_{P_{2i}}}{T_{P_{2i}}} 0 0 0 \right]^T \), where \( (2, 3)_1 = (2, 3)_3 = \frac{1}{T_{G_{1i}}} \), \( (2, 3)_2 = (2, 3)_4 = \frac{a_1}{T_{G_{1i}}} \), \( (7, 8)_1 = (7, 8)_2 = \frac{1}{T_{G_{2i}}} \), \( (7, 8)_3 = (7, 8)_4 = \frac{a_2}{T_{G_{2i}}} \). The parameters \( \frac{1}{T_{G_{ki}}} \) and \( \frac{1}{r_k T_{G_{ki}}} \), \( k \in F_2 \), are assumed to be unknown, time-varying but bounded within 5% of their nominal values.

4. Robust fuzzy-model-based LFC design

This section discusses control design conditions so that the objective of LFC can be successfully achieved.

Consider the uncertain T–S fuzzy system represented by

\[
\dot{x}(t) = \sum_{i=1}^{q} \theta_i(z(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)) \tag{10}
\]

**Remark 2.** The system (10) is the version in which \( w(t) \) is set to zero in (4). From Assumption 2, one can expect that \( w(t) \) does not affect the stability of the controlled system in the sense of Lyapunov. That is, the asymptotic stability of (10) means the boundedness of the solution to (4). This relationship will be rigorously explored later using the uniformly ultimately boundedness (UUB).

Consider a fuzzy-model-based control in the form:

\[
u(t) = \sum_{i=1}^{q} \theta_i(z(t))K_i x(t) \tag{11}
\]

The closed-loop system of (10) and (11) can be written as

\[
\dot{x}(t) = \sum_{i=1}^{q} \theta_i^2(z(t))(A_i + \Delta A_i) + (B_i + \Delta B_i)K_i)x(t) + 2 \sum_{i<j}^{q} \theta_i(z(t))\theta_j(z(t))
\times \left( A_i + \Delta A_i + (B_i + \Delta B_i)K_j + A_j + \Delta A_j + (B_j + \Delta B_j)K_i \right)x(t).
\tag{12}

Define the Lyapunov functional candidate as

\[
V(x(t)) = x(t)^TPx(t) \tag{13}
\]

where \( P = P^T > 0 \). Clearly, \( V(0) = 0 \), \( V(x(t)) > 0 \), and radially unbounded in any neighborhood of \( x_{eq} = [0]_{n 	imes 1} \). Considering Assumption 1, the time derivative of \( V(x(t)) \) along the trajectory of (12) is computed by
\[ \dot{V}(x(t)) = \sum_{i=1}^{q} \theta_i^2(x(t))x(t)^T \left((A_i + H_iF_i(t)E_{ai} + (B_i + H_iF_i(t)E_{bi})K_i)\right)^TP \\
+ P(A_i + H_iF_i(t)E_{ai} + (B_i + H_iF_i(t)E_{bi})K_i) + \gamma P - \gamma P)x(t) \\
+ 2\sum_{i<j}^{q} \theta_i(z(t))\theta_j(z(t))x(t)^T \\
\times \left(\frac{(A_i + H_iF_i(t)E_{ai} + (B_i + H_iF_i(t)E_{bi})K_i)}{2}\right)^T \\
\times \frac{(A_j + H_jF_j(t)E_{aj} + (B_j + H_jF_j(t)E_{bj})K_j)}{2} \\
\times P(A_j + H_jF_j(t)E_{aj} + (B_j + H_jF_j(t)E_{bj})K_j) + \gamma P - \gamma P)\right)x(t) \]

(14)

where \( \gamma \in \mathbb{R}_{>0} \) is a given scalar. Now, we get the following result.

**Theorem 1.** If there exist a matrix \( W = W^T > 0 \), and matrices \( M_i \), and some scalars \( \epsilon_{ij} \in \mathbb{R}_{>0} \), such that the following linear matrix inequalities (LMIs) are satisfied, then (10) is robustly asymptotically stabilizable on \( U \) in the sense of Lyapunov, by employing (11), in the presence of the parametric uncertainties:

\[
\begin{bmatrix}
\Psi_i + \epsilon_{ii}H_iH_i^T \quad (\bullet)^T \\
E_{ai}W + E_{bi}M_i - \epsilon_{ii}I
\end{bmatrix} < 0, \quad i \in \mathcal{J}_Q \nn\
\begin{bmatrix}
\mathcal{Y}_{ij} + \epsilon_{ij}H_iH_i^T + \epsilon_{ij}H_jH_j^T \quad (\bullet)^T \quad (\bullet)^T \\
E_{ai}W + E_{bi}M_j - \epsilon_{ij}I \nn\end{bmatrix} < 0, \quad (i, j) \in \mathcal{J}_J \times \mathcal{J}_Q 
\]

(15)

(16)

where \( \Psi_i = WA_i^T + A_iW + M_i^TB_i^T + B_iM_i + \gamma W \), \( \mathcal{Y}_{ij} = WA_i^T + A_iW + WA_j^T + A_jW + M_j^TB_j^T + B_jM_j + M_i^TB_i + B_iM_i + 2\gamma W \), \( W = P^{-1} \) and \( M_i = K_iP^{-1} \), and \( (\bullet)^T \) denotes the transposed element in symmetric positions and \( \mathcal{J}_J \times \mathcal{J}_Q \) means all pairs \( (i, j) \in \mathbb{Z}_{>0} \times \mathbb{Z}_{>0} \) such that \( 1 \leq i \leq j \leq r \).

**Proof.** See Appendix A.1. \( \square \)

**Remark 3.** Theorem 1 only guarantees the asymptotic stability of (10). In our problem formulation of LFC, \( \Delta P_{D_i}(t) \) exists, which means that the origin \( x(t) = [0]_{n \times 1} \) may not be an equilibrium point of the perturbed system (4). Thus, if we use Theorem 1 in designing the LFC, the asymptotic stability...
should not be expected. All one can hope for is to study whether \( x(t) \) will be bounded within a certain region. Theorem 2 ensures us that the closed-loop solution to (4) and (11) will be remained in an arbitrary compact set and UUB. This fact justifies the use of control law based on Theorem 1 for the LFC objective.

**Theorem 2.** Consider the closed-loop T–S fuzzy system (4) with (11) that ensures the asymptotic stability of (12). If the following conditions are satisfied, then the perturbed system (4) controlled via (11) is UUB.

(1) Assumption 2 holds.
(2) Theorem 1 holds.
(3) Existence of a compact set \( \Theta \) such that \( x(0) \in \Theta \).

**Proof.** The proof is given in Appendix A.2. \( \square \)

**Remark 4.** In conclusion, recalling Assumption 2 and the power system with the valve position limits (5), Theorems 1 and 2 suffice to guarantee that the system variables are robustly UUB against the parametric uncertainties as well as the nonlinearity. Moreover, the asymptotic convergence of \( \Delta f_k(t) \) can be accomplished via (11) with the aid of the integral control (6).

5. An example

This section presents a robust LFC design example for the two-area power system with valve position limits. The control objective is to drive \( \Delta f_1(t) \) and \( \Delta f_2(t) \) to zero. The nominal values of system parameters are shown in Table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Area 1</th>
<th>Area 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Area 1</td>
<td>Area 2</td>
</tr>
<tr>
<td>( T_{p_i} )</td>
<td>20 s</td>
<td>20 s</td>
</tr>
<tr>
<td>( T_{T_i} )</td>
<td>0.3 s</td>
<td>0.2 s</td>
</tr>
<tr>
<td>( T_{G_i} )</td>
<td>0.08 s</td>
<td>0.1 s</td>
</tr>
<tr>
<td>( K_{P_i} )</td>
<td>120 Hz/p.u. MW</td>
<td>125 Hz/p.u. MW</td>
</tr>
<tr>
<td>( R_k )</td>
<td>2.4 Hz/p.u. MW</td>
<td>2.4 Hz/p.u. MW</td>
</tr>
<tr>
<td>( K_{E_i} )</td>
<td>0.423 p.u. MW</td>
<td>0.5 p.u. MW</td>
</tr>
<tr>
<td>( T_{12} )</td>
<td>0.545 p.u. MW</td>
<td>0.545 p.u. MW</td>
</tr>
</tbody>
</table>
The parameters \( \Delta X_{GM_k} \) and \( \Delta X_{Gm_k} \), \( k \in \mathcal{I}_2 \), are assumed as 0.12, -0.03 p.u., respectively. The design parameters \( \alpha_1 \) and \( \alpha_2 \) are arbitrarily chosen as 0.47 such that the premise variables belong to their universes of discourses, \([\Omega^L_1, \Omega^U_1] \) and \([\Omega^L_2, \Omega^U_2] \) for all \( t \in \mathbb{R}_{\geq 0} \). Based on Assumption 1, the uncertain matrices \( \Delta A_i \), \( \Delta B_i \) and \( \Delta D_i \) are decomposed as follows:

\[
H_i = 0.6708 \times \begin{bmatrix}
0 & 0 & -\frac{1}{T_{G_i}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{T_{G_i}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{G_2}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{G_2}}
\end{bmatrix}_i
\]

\[
E_{a_i} = 0.0745 \times \begin{bmatrix}
\frac{1}{R_{1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0
\end{bmatrix}_i
\]

\[
E_{b_i} = 0.0745 \times \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}_i
\]

\[E_{d_i} = [0]_{4 \times 1}\]

where \( i \in \mathcal{I}_4 \). By applying Theorem 1 and solving the associated LMIs, we obtain the following control gain matrices for (11):

\[
K_1 = \begin{bmatrix}
\end{bmatrix}
\]

\[
K_2 = \begin{bmatrix}
-4.763 & -5.969 & -0.384 & -18.285 & 4.344 & -0.635 & -0.547 & -0.338 & 0.909 \\
\end{bmatrix}
\]

\[
K_3 = \begin{bmatrix}
-0.158 & -0.377 & -0.185 & 4.429 & -4.801 & -5.335 & 5.390 & -1.087 & -18.878
\end{bmatrix}
\]

\[
K_4 = \begin{bmatrix}
-5.321 & -6.570 & -0.572 & -19.519 & 4.051 & -1.633 & -1.432 & -0.653 & -2.352 \\
-1.158 & -1.444 & -0.421 & 1.029 & -4.506 & -5.995 & -5.910 & -1.375 & -20.557
\end{bmatrix}
\]

The initial state is \( x(0) = [0]_9 \times 1 \). The system parameters \( \frac{1}{T_{G_k}} \) and \( \frac{1}{R_{k}T_{G_k}} \), \( k \in \mathcal{I}_2 \), are being randomly varied within 5% of their nominal values during the simulation. The simulation time is 10 s. We assume that the power system experiences the large load change \( \Delta P_{D_2}(t) = 0.1, t \in \mathbb{R}_{\geq 0} \) on Area 2 and \( \Delta P_{D_2}(t) = 0.1, t \in \mathbb{R}_{> 4} \) on Area 1, due to the deregulated generation environment.

For comparison purpose, two other LFC methods—LFC scheme based on the linear quadratic (LQ) control [1] and the conventional integral LFC, are simulated. It is noted that the integral LFC implies (6) with the associated parameters in Table 2 and \( \Delta P_{C_k}(t) = 0 \) for \( t \in \mathbb{R}_{\geq 0}, k \in \{1, 2\} \). The LQ method is applied to the linearized model without considering the valve position limit
nor the parameter uncertainties. That is, the standard LQ control technique utilizes one pair \((A_1, B_1)\) from our T–S fuzzy model for nonlinear power systems and the weighting matrices [1]

\[
Q = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & T_{12}^2 + 1 & 0 & 0 & 0 & -T_{12}^2 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -T_{12}^2 & 0 & 0 & 0 & T_{12}^2 + 1
\end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

to produce

\[
K_{LQ} = \begin{bmatrix} -0.753 & -1.403 & -0.657 & -0.511 & 1.370 & -0.076 & -0.020 & -0.001 & 0.085 \\ 0.040 & 0.011 & -0.001 & 0.110 & -0.701 & -0.867 & -0.962 & -0.400 & -0.513 \end{bmatrix}
\]

All simulations are performed with the uncertain nonlinear power system model (7). Figs. 4 and 5 report the time responses of \(\Delta f_1(t)\) and \(\Delta f_2(t)\) by the compared LFC schemes and the proposed method. Although \(\frac{1}{T_0}\) and \(\frac{1}{R_k T_0}\) are randomly varied during the simulation process and the valve position limits is considered, the trajectories of \(\Delta f_1(t)\) and \(\Delta f_2(t)\) by the proposed method converge to zero asymptotically, furthermore, its time responses are better, in terms of overshoots and settling time, than the compared methods.

However, as shown in the figures, one can immediately recognize that the LQ method does not directly steer \(\Delta f_k(t)\) to zero, but gives rise to oscillations.

Fig. 4. Time responses of \(\Delta f_1(t)\): proposed (solid), LQ (dashed), and integral (dotted), where \(\Delta P_{D_1}(t) = 0.1\) p.u., \(t \in \mathbb{R}_{>4}\), \(\Delta P_{D_2}(t) = 0.1\) p.u., \(t \in \mathbb{R}_{>0}\).
The worst case is the integral control, which fails in LFC objective. The reason can be explained as follows: $\Delta X_{G_k}(t)$ in each area generally supplies an additional power $\Delta P_{G_k}(t)$ to compensate for each $\Delta P_{D_k}(t)$. Notably, it is observed from Fig. 9 that $\Delta X_{G_2}(t)$ suffers from severe valve position limits after approximately $t = 4.6$ s. Such nonlinearity has not been theoretically considered in the compared techniques. As a result, the local generator in Area 2 by the existing methods does not fully compensate for the load disturbance $\Delta P_{D_2}(t)$ but produces fluctuations on $\Delta P_{G_2}(t)$ and the degraded LFC performances on $\Delta f_2(t)$. This imperfect LFC influences on Area 1 via the tie line, which are illustrated in Figs. 6–9. On the other hand, $\Delta X_{G_2}(t)$ by the proposed method are rapidly guided to 0.1, and hence $\Delta P_{G_2}(t)$ well compensate for $\Delta P_{D_2}(t)$.

When a large load change occurs due to the deregulated generation environment, the integral control technique generates a large control signal and $\Delta X_{G_k}(t)$ is restricted by the valve position limit. The control (6) continues to

![Fig. 5. Time responses of $\Delta f_2(t)$: proposed (solid), LQ (dashed), and integral (dotted), where $\Delta P_{D_1}(t) = 0.1$ p.u., $t \in R_{>4}$, $\Delta P_{D_2}(t) = 0.1$ p.u., $t \in R_{>0}$.](image)

![Fig. 6. Time responses of $\Delta P_{G_1}(t)$: proposed (solid), LQ (dashed), and integral (dotted), where $\Delta P_{D_1}(t) = 0.1$ p.u., $t \in R_{>4}$, $\Delta P_{D_2}(t) = 0.1$ p.u., $t \in R_{>0}$.](image)
Fig. 7. Time responses of $\Delta P_{G_1}(t)$: proposed (solid), LQ (dashed), and integral (dotted), where $\Delta P_{D_1}(t) = 0.1 \text{ p.u.}, t \in \mathbb{R}_{>4}, \Delta P_{D_2}(t) = 0.1 \text{ p.u.}, t \in \mathbb{R}_{>0}$.

Fig. 8. Time responses of $\Delta X_{C_1}(t)$: proposed (solid), LQ (dashed), and integral (dotted), where $\Delta P_{D_1}(t) = 0.1 \text{ p.u.}, t \in \mathbb{R}_{>4}, \Delta P_{D_2}(t) = 0.1 \text{ p.u.}, t \in \mathbb{R}_{>0}$.

Fig. 9. Time responses of $\Delta X_{C_2}(t)$: proposed (solid), LQ (dashed), and integral (dotted), where $\Delta P_{D_1}(t) = 0.1 \text{ p.u.}, t \in \mathbb{R}_{>4}, \Delta P_{D_2}(t) = 0.1 \text{ p.u.}, t \in \mathbb{R}_{>0}$. 
integrate the frequency deviation, thus produces ever larger control signal and
\( \Delta X_{Gk}(t) \) is locked in the valve position limit. This results in large overshoot and
long settling time. The deteriorated performance by the LQ technique can be
simply explained by the mismatch between the models used in design and sim-
ulation. On the other hand, the proposed LFC is designed to theoretically cope
with the valve position nonlinearity and the parametric uncertainties, its good
control performance is not unexpected. In view of the simulation results, it can
be claimed that the proposed control has strong robustness against the para-
metric uncertainties and the valve position limits.

6. Concluding remarks

In this paper, a systematic LFC design methodology has been proposed for
uncertain nonlinear power systems. The typical nonlinearity—the valve posi-
tion limit on the governor and the parametric uncertainties are concerned,
which are important under the deregulated operation of power systems. The
designed fuzzy-model-based LFC provides an robust control performance with
the asymptotic convergence of the incremental frequency deviation and the
UUB of the solution to the whole dynamical power system. The simulation
results have highly visualized the excellence of the developed LFC scheme,
which convincingly points out its great applicability to the electric power
industry.

Acknowledgment

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Appendix A

A.1. Proof of Theorem 1

Before proceeding further, recall the following lemma.

**Lemma 1.** Given constant matrices \( H, E, \) and \( F \) with \( F^TF \leq I \), and a matrix
\( S = S^T \) of appropriate dimensions, the following inequalities are equivalent

\[
\begin{align*}
(1) \quad S + HFE + E^TF^TH^T & < 0 \\
(2) \quad S + \begin{bmatrix} \epsilon^{-1}E^T & \epsilon H \\ \epsilon H^T & \epsilon \end{bmatrix} \begin{bmatrix} \epsilon^{-1}E \\ \epsilon H^T \end{bmatrix} & < 0 \quad \text{for some } \epsilon \in \mathbb{R}_{>0}.
\end{align*}
\]
If (14) is negative definite uniformly for all $x(t)$ and for all $t \geq 0$ except at $x(t) = 0$, then (12) is asymptotically stable about its zero equilibrium. Therefore, it suffices to force the two sums in (14) to be negative definite, respectively, for the robustly asymptotic stability on $U$ of (12). First, assume that the following inequality in the first sum of (14) is negative definite,

$$\Phi_{ii} + PH_iF_i(t)(E_{ai} + E_{bi}K_i) + (E_{ai} + E_{bi}K_i)^TH_i^TP < 0$$

(A.1)

where $\Phi_{ii} = A_i^TP + PA_i + K_i^TB_i^TP + PB_iK_i + \gamma P$. According to Lemma 1, for some $\epsilon_i > 0$, (A.1) is equivalent to

$$\Phi_{ii} + \left[(E_{ai} + E_{bi}K_i)^T PH_i\right] \begin{bmatrix} \epsilon_i^{-1}I & (\bullet)^T \\ 0 & \epsilon_iI \end{bmatrix} \left[ E_{ai} + E_{bi}K_i \right] (PH_i)^T < 0$$

(A.2)

If one applies the Schur complement to (A.2) and takes a congruence transformation, then the change of variables $W = P^{-1}$ and $M_i = K_iP^{-1}$ yield the first LMI in Theorem 1. The second LMI can be also established through a similar technique and procedure. This completes the proof of the theorem.

A.2. Proof of Theorem 2

Consider the Lyapunov functional candidate (13). Differentiating (13) with respect to time and along the trajectory of (4) yields

$$\dot{V}(x(t)) = \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t))\theta_j(z(t))(x(t))^T((A_i + \Delta A_i $$

$$+ (B_i + \Delta B_i)K_j)^TP + P((A_i + \Delta A_i + (B_i + \Delta B_i)K_j)x(t)$$

$$+ w(t)^T(D_i + \Delta D_i)^TPx(t) + x(t)^TP(D_i + \Delta D_i)w(t))$$

$$\leq \sum_{i=1}^{q} \theta_i(z(t))(-\gamma x(t)^TPx(t) + w(t)^T(D_i + H_iF_i(t)E_{di})^TPx(t)$$

$$+ x(t)^TP(D_i + H_iF_i(t)E_{di})w(t))$$

(A.3)

Using Lemma 1 and Assumption 2, One obtain

$$\dot{V}(x(t)) \leq \sum_{i=1}^{q} \theta_i(z(t))\left(x(t)^T(-\gamma P + \frac{1}{\xi_1}PD_iD_i^TP + \frac{1}{\xi_2}PH_iH_i^TP)x(t)$$

$$+ w(t)^T(\xi_1I + \xi_2E_{di}^TE_{di})w(t) \right)$$

$$\leq -v_1\|x(t)\|^2 + v_2\|w(t)\|^2 \leq -v_1\|x(t)\|^2 + v_2\xi_2^2$$

where $v_1 = \min\left(\lambda_{\min}\left(\gamma P - \frac{1}{\xi_1}PD_iD_i^TP - \frac{1}{\xi_2}PH_iH_i^TP\right)\right)$, $v_2 = \max(\lambda_{\max}(\xi_1I + \xi_2E_{di}^TE_{di}))$, and $\xi_1$ and $\xi_2$ are arbitrary chosen such that $v_1$ is positive. Define
a set \( \Pi = \left\{ x(t) \in \mathbb{R}^n \left| \| x(t) \| \leq \frac{n}{n-1} \zeta^2 \right. \right\} \) which is obviously compact. One can easily agree that \( \dot{V}(x(t)) \) is negative as long as \( x(t) \) is outside \( \Pi \). According to the standard Lyapunov theorem [10], we conclude the controlled state \( x(t) \) of (4) via (11) is bounded and will converge to \( \Pi \). Furthermore, If we initialize \( x(0) \) inside the compact set \( \Theta \), there exist a constant \( T \) such that all trajectories will converge to \( \Pi \) and remain in \( \Pi \) for all time \( t > T \). This implies that the closed-loop system is UUB.

References